A Comparative Study on Paradoxical Conditionals in Classical Mathematical Logic and Strong Relevant Logics

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Abstract

From the viewpoint of logic, we have qualitatively shown that classical mathematical logic, its various classical conservative extensions, and traditional relevant logics are not suitable to underlying forward reasoning and/or deduction for discovery because their logical theorems include a lot of paradoxes of conditional, and shown that strong relevant logics are more hopeful candidates for the purpose. But, it is not clear that how 'bad' the classical mathematical logic, its various classical conservative extensions, and traditional relevant logics are, and how 'good' strong relevant logics are, since no quantitative analysis and discussion is reported until now. As the result of a comparative study, in this paper, we present a quantitative analysis and discussion on paradoxical conditionals in classical mathematical logic and strong relevant logics. Our study shows that the problem of paradox is very critical to the development of forward reasoning and/or deduction engines useful in applications in the real world.

1. Introduction

Forward reasoning and/or deduction based on some fundamental logic system is indispensable to any computing system to discover new knowledge or predict future incidents. In principle, any logic system can be used as a fundamental logic to underlie forward reasoning and/or deduction processes with a certain purpose, if the logical consequence relation defined by the logic system is correspond or suitable to the purpose of the reasoning and/or deduction processes. However, from the viewpoint of practice, only those logic systems, which define logical consequence relations correspond or suitable to the purposes of reasoning and/or deduction processes required by problem solving in the real world, should be used as the underlying logic systems for the forward reasoning and/or deduction processes.

From the viewpoint of logic, we have qualitatively shown that classical mathematical logic (CML for short), its various classical conservative extensions, and traditional relevant logics (RL for short) are not suitable to underlying forward reasoning and/or deduction from discovery because their logical theorems include a lot of paradoxes of conditional, and shown that strong relevant logics (SRL for short) are more hopeful candidates for the purpose [3, 4, 5]. But, it is not clear that how 'bad' the classical mathematical logic, its various classical conservative extensions, and traditional relevant logics are, and how 'good' strong relevant logics are, since no quantitative analysis and discussion is reported until now. As the result of a comparative study, in this paper, we present a quantitative analysis and discussion on paradoxical conditionals in CML and SRL. Our study shows that the problem of paradox is very critical to the development of forward reasoning and/or deduction engines useful in applications in the real world.

The rest of this paper is organized as follows: Section 2 gives a very simple introduction to the qualitative analysis on CML and SRL in scientific reasoning as well as our everyday reasoning. Section 3 gives the quantitative analysis on paradoxical conditionals in CML and SRL. Section 4 discusses about our comparative study. Some concluding remarks are given in Section 5.

2. Forward reasoning based on logics

Reasoning is the process of drawing new conclusions from given premises, which are already known facts or previously assumed hypotheses (Note that how to define the notion of "new" formally and satisfactorily is still a difficult open problem until now). The validness of reasoning is a matter of the connection between its premises and its conclusion, and concerns the strength of the relation between them. What is the criterion by which one can decide whether the conclusion of a reasoning really does follow from its premises or not? Is there the only one criterion, or are there many criteria? If there are many criteria, what are the intrinsic differences between them? It is logic that deals with the validity of argument and reasoning in general.

What is logic? Logic is a special discipline which is considered to be the basis for all other sciences, and therefore, it is a science prior to all others, which contains the ideas and principles underlying all sciences [9, 13]. Logic deals with what entails what or what follows from what, and aims at determining which are the correct conclusions of a given set of premises, i.e., to determine which arguments are valid. Therefore, the most essential and central concept in logic is the logical consequence relation that relates a given set of premises to those conclusions, which validly follow from the premises.

IF-THEN rules have played and are still playing various important roles in mathematical, natural, social and human sciences. Scientists always use conditionals in their descriptions of various definitions, propositions, and theorems to connect a concept, fact, situation or conclusion to its sufficient conditions. The major work of almost all scientists is to discover some sufficient condition relations between various phenomena, data, and laws in their research fields. In logic, a sentence in the form of "if . . . then . . ." is usually called a conditional proposition or simply conditional. A conditional must concern two parts which are connected by the connective "if . . . then . . ." and called the antecedent and the consequent of that conditional. The truth of a conditional depends not only on the truth of its antecedent and consequent but also, and more essentially, on a necessarily relevant and/or conditional relation between its antecedent and consequent.

From the viewpoint of object logic (i.e., the logic we are studying), there are two classes of conditionals. One class is empirical conditionals and the other class is logical conditionals. In the sense of logic, an empirical conditional is that its truth-value is depend on the contents of its antecedent and consequent. Therefore empirical conditionals cannot be determined only by its abstract form. A logical conditional is that its truth-value is universally true or false and therefore can be determined by its abstract form. A logical conditional that is considered to be universally true, in the sense of that logic, is also called entailment of that logic [1, 2, 7]. The most intrinsic difference between various different logic systems is to regard what class of conditionals as entailments.

2.1. Classical mathematical logic and its various extensions

Classical mathematical logic (CML for short) was established in order to provide formal languages for describing the structures with which mathematicians work, and the methods of proof available to them; its principal aim is a precise and adequate understanding of the notion of mathematical proof.

In CML, the notion of conditional, which is intrinsically intensional but not truth-functional, is represented by the truth-functional extensional notion of material implication (denoted by $\rightarrow$ in this paper) that is defined as $A \rightarrow B =_{df} \neg(A \wedge \neg B)$ or $A \rightarrow B =_{df} \neg A \vee B$. However, the material implication is intrinsically different from the notion of conditional in meaning (semantics). It is no more than an extensional truth-function of its antecedent and consequent but does not require that there is a necessarily relevant and conditional relation between its antecedent and consequent, i.e., the truth-value of the formula $A \rightarrow B$ depends only on the truth-values of A and B, though there could exist no necessarily relevant and conditional relation between A and B. It is this intrinsic difference in meaning between the notion of material implication and the notion of conditional that leads to the well-known "implicational paradox problem" in CML. The problem is that if one regards the material implication as the notion of conditional and regards every logical theorem of CML as an entailment or valid reasoning form, then a great number of logical axioms and logical theorems of CML, such as $A \rightarrow (B \rightarrow A)$, $B \rightarrow (\neg A \vee A)$, and so on, present some paradoxical properties and therefore they have been referred to in the literature as "implicational paradoxes" [1, 2, 8, 11, 12]. Note that any classical conservative extension has the similar problems as the above problems in CML [5].

Consequently, in the framework of CML, its various classical conservative extensions, even if a reasoning is valid in the sense of CML, neither the necessary relevance between its premises and conclusion nor the truth of its conclusion in the sense of conditional can be guaranteed necessarily.

2.2. Strong relevant logics

Traditional relevant (or relevance) logics ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called "paradoxes" of material and strict implication [1, 2, 8, 11, 12]. Some major traditional relevant logic systems are "system E of entailment", "system R of relevant implication", and "system T of ticket entailment". Anderson and Belnap proposed variable-sharing as a necessary but not sufficient formal condition for the relevance between the antecedent and consequent of an entailment. The underlying principle of these relevant logics is the relevance principle, i.e., for any entailment provable in E, R, or T, its antecedent and consequent must share a sentential variable. Variable-sharing is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment [1, 2, 8, 11, 12]. It is this relevance principle that excludes those implicational paradoxes from logical axioms or theorems of relevant logics.

However, although the traditional relevant logics have rejected those implicational paradoxes, there still exist some logical axioms or theorems in the logics, which are not so natural in the sense of conditional. Such logical axioms or theorems, for instance, are $(A \wedge B) \Rightarrow A$, $A \Rightarrow (A \vee B)$, and so on, where $\Rightarrow$ denotes the primitive intensional connective in the logics to represent the notion of conditional. Cheng named these logical axioms or theorems 'conjunction-implicational paradoxes' and 'disjunction-implicational paradoxes' [5].

In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present we has proposed some strong relevant (relevance) logics, named Rc, Ec, and Tc [5, 6]. The logics require that the premises of an argument represented by a conditional include no unnecessary and needless conjuncts and the conclusion of that argument includes no unnecessary and needless disjuncts. As a modification of traditional relevant logics R, E, and T, strong relevant logics Rc, Ec, and Tc rejects all conjunction-implicational paradoxes and disjunction-implicational paradoxes in R, E, and T, respectively. Since the strong relevant logics are free of not only implicational paradoxes but also conjunction-implicational and disjunction-implicational paradoxes, in the framework of strong relevant logics, if a reasoning is valid, then both the relevance between its premises and its conclusion and the validity of its conclusion in the sense of conditional can be guaranteed in a certain sense of strong relevance.

2.3. Terminology

For a formal logic system where the notion of conditional is represented by a primitive connective "$\Rightarrow$", a formula is called a zero degree formula if and only if there is no occurrence of "$\Rightarrow$" in it; a formula of the form "$A \Rightarrow B$" is called a first degree conditional if and only if both A and B are zero degree formula; a formula A is called a first degree formula if and only if it satisfies the one of the following conditions: (1) A is a first degree conditional, (2) A is in the form $+B$ ($+$ is a one-place connective such as negation and so on) where B is a first degree formula, (3) A is in the form $B * C$, ($*$ is a non-implicational two-place connective such as conjunction or disjunction and so on), where both of B and C is a first degree formulas, or one of B and C is a first degree formula and the another is a zero degree formula. Let k be a natural number. A formula of the form "$A \Rightarrow B$" is called a k^{th} degree conditional if and only if both A and B are $(k-1)^{th}$ degree formulas, or either formula A or B is a $(k-1)^{th}$ degree formula and the another is a $j^{th}(j < k-1)$ degree formula; a formula is called k^{th} degree formula if and only if it satisfies the one of the following conditions: (1) A is a k^{th} degree conditional, (2) A is in the form $+B$ ($+$ is a one-place connective such as negation and so on) where B is a k^{th} degree formula, (3) A is in the form $B * C$, ($*$ is a non-implicational two-place connective such as conjunction or disjunction and so on), where both of B and C is a k^{th} degree formulas, or one of B and C is a k^{th} degree formula and the another is a $j^{th}(j < k)$ degree formula.

3. The comparative study of paradoxical conditionals

Now, we do a quantitative analysis on the roles and effectiveness of CML and SRL in forward reasoning and/or deduction by investigating the number of paradoxical conditionals in them. We compare the kinds of schemata of well-formed conditionals, i.e., well-formed formulas with only connective to represent the notion of conditional (wfc for short), with the kinds of schemata of wfc satisfied the strong relevance principle.

The strong relevance principle (SRP for short) is the one of principles in strong relevant logics: if A is a theorem of Rc, Ec, or Tc, then every sentential variable in A occurs at least once as an antecedent part and at least once as a consequent part. The definition of an antecedent part and a consequent part is as follows, let A, B and C be well-formed formulas,

1. A is a consequent part of A,

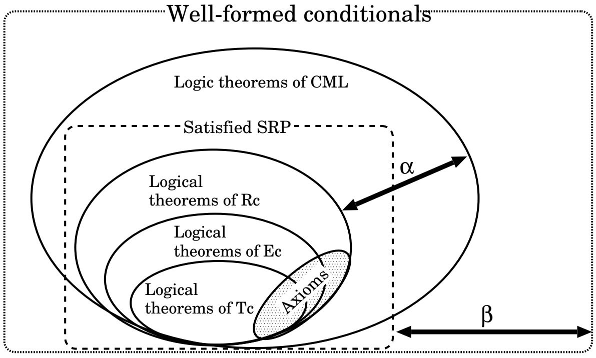

Figure 1. The set of well-formed conditionals

2. if $\neg B$ is a consequent part (antecedent part) of A, then B is an antecedent part (consequent part) of A,

3. if $B \Rightarrow C$ is a consequent part (antecedent part) of A, then B is an antecedent part (consequent part) of A, and C is consequent (antecedent part) of A,

4. if $B \wedge C$ or $B \vee C$ is a consequent part (antecedent part) of A, then both B and C are consequent parts (antecedent parts) of A.

Figure 1 shows the relationships among the set of logical theorems of conditional in CML, the set of wfc satisfied SRP and the set of logical theorems of conditional in Rc, Ec, and Tc. Note that SRL has not only entailment $\Rightarrow$ as a primitive logical connective, but also material implication $\rightarrow$ as a defined logical connective [5]. Actually, in viewpoint from syntax, the relationship between well-formed formulas of SRL, denoted by WFF_{SRL}, and that of CML, denoted by WFF_{CML}, is

$$\mathrm{WFF}_{CML} \subset \mathrm{WFF}_{SRL} \quad (1)$$

However, in this paper, we regard material implication $\rightarrow$ in CML and entailment $\Rightarrow$ in SRL as a same connective to represent the notion of conditional.

In a normal situation, to investigate the number of paradoxical conditionals in CML, we should compare the kinds of the logical theorem schemata of conditionals in CML with the kinds of the logical theorem schemata of conditionals in Rc to obtain the number of elements of set α in Fig. 1. However, it is difficult, if not impossible, to get the correct number of kinds of logical theorem schemata of conditionals in CML, Rc, Ec, and Tc.

On the other hand, to obtain the number of kinds of schemata of wfc and that of wfc satisfied SRP are possible. If the difference between the number of kinds

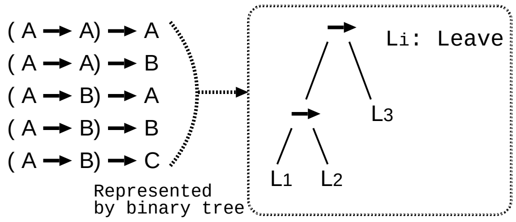

Figure 2. The conditional schema represented by binary tree

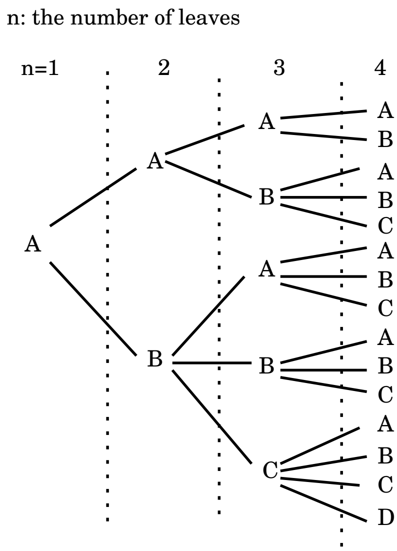

Figure 3. The combination of sentential variables

of logical theorem schemata of CML and that of Rc, i. e. , α in Fig. 1, and the difference between the number of kinds of schemata of wfc and that of wfc satisfied SRP, i. e. , β in Fig. 1, have the same tendency, then it is possible to analogize the number of paradoxical conditionals in CML by comparing the kinds of schemata of wfc with that of wfc satisfied SRP.

We calculate the number of kinds of k^{th} degree schemata of wfc and k^{th} degree schemata of wfc satisfied SRP.

The calculation of the number of kinds of k^{th} degree schemata of wfc accords to following processes.

1. Making all kinds of binary trees from depth 1 to depth k, by which the schemata of wfc without sentential variables are represented as the node is the connective to represent the notion of conditional like Fig. 2.

2. Classifying the binary trees made at above process by the number of leaves.

3. Calculating the combination of sentential variables of the number of leaves n. For example, if the number of leaves is 4, then the combination of sentential variables is like Fig. 3.

4. Calculating the number of kinds of k^{th} degree schemata of wfc with above results. $B(n)$ denotes the kinds of schemata without sentential variables of wfc that the number of leaves is n. $C(n)$ is the combination of sentential variables that the number of leaves is n. $S_{\text{wfc}}(k)$ denotes the number of kinds of k^{th} degree schemata of wfc.

$$S_{\text{wfc}}(k) = \sum_{n=1}^{2^{k+1}} B(n) \cdot C(n) \qquad (2)$$

The calculation of the number of kinds of k^{th} degree schemata of wfc satisfied SRP accords to following processes.

1. Making all kinds of binary trees from depth 1 to depth k, by which the schemata of wfc without sentential variables are represented as the node is the connective to represent the notion of conditional like Fig. 2.

2. Making the combination of antecedent parts and consequent parts of leaves in each binary tree .

3. Classifying the combination of antecedent parts and consequent parts by the kinds of the combinations.

4. Making all combinations of sentential variables of the length n.

5. Choosing the combinations of sentential variables satisfied SRP in each combination of antecedent parts and consequent parts which is the number of leaves n.

6. Calculating the number of kinds of k^{th} degree schemata of wfc satisfied SRP with above results. Let i denote the unique number of the combination of antecedent parts and consequent parts made at process 2, ($1 \leq i \leq$ Max — Max denotes the number of the combinations of antecedent parts and consequent parts). $B_{SRP}(i)$ denotes the number of binary trees whose combination of antecedent parts and consequent parts is i-th. $C_{SRP}(i)$ is the combinations of sentential variables satisfied SRP in i-th combination of antecedent parts and consequent parts. $S_{SRP}(k)$ denotes the number of kinds of k^{th} degree schemata of wfc satisfied SRP.

$$S_{SRP}(k) = \sum_{i=1}^{Max} B_{SRP}(i) \cdot C_{SRP}(i) \qquad (3)$$

Table 1. The number of kinds of schemata of wfc and that of wfc satisfied SRP

degree	wfc (A)	wfc satisfied SRP(B)	$\frac{B}{A}$
1	2	1	1/2
2	25	6	1/4
3	1.1×10^4	9.9×10^2	1/10
4	3.9×10^{10}	1.1×10^{10}	1/4
5	7.1×10^{26}	1.3×10^{24}	1/500

Table 2. The number of kinds of logical theorem schemata of CML$_\rightarrow$ and Rc$_\Rightarrow$

degree	CML$_\rightarrow$	Rc$_\Rightarrow$	$\frac{\text{Rc}_\Rightarrow}{\text{CML}_\rightarrow}$
1	1	1	1
2	8	3	1/2
3	277	20	1/13

Table 1 shows the number of kinds of k^{th} $(1 \leq k \leq 5)$ degree schemata of wfc and that of wfc satisfied SRP. It shows the number of kinds of schemata of wfc is the from 4 to 500 times more than the number of kinds of schemata of wfc satisfied SRP.

Table 2 shows the number of kinds of k^{th} degree logical conditional theorem schemata of CML and that of Rc which are deduced by an automated forward deduction system for general-purpose entailment calculus, named EnCal [4]. EnCal supports forward entailment calculi based on strong relevant logics [3] as well as other logics. At present, EnCal has not deduced more than 4th degree logical conditional theorem of CML yet, since the execution time and the amount of main-memory needed of the deduction in EnCal is large. Table 2 shows the number of kinds of logical conditional theorem schemata of CML is from 2 times to 13 times as many as that of Rc.

4. Discussion

From table 1 and 2, our experiments show the difference between the number of kinds of schemata of wfc and that of wfc satisfied SRP and the difference between the number of kinds of logical theorem schemata of conditional in CML and that in Rc have a same tendency to increase the difference as the degree of nested conditionals increases. The tendency comes from the difference of the kinds of sentential variable in a formula.

From Eq. (2), $S_{\text{wfc}}(k)$, the number of kinds of k^{th} degree schemata of wfc, is calculated from the kinds of binary trees from depth 1 to k, by which conditional schemata without sentential variables are represented as the node is the connective to represent the notion of conditional, and the combination of sentential variables of the number of leaves n $(1 < n \leq 2^{k-1})$. $B(n)$ denotes the number of kinds of the binary trees from depth 1 to k that the number of leaves is n. $C_i(n)$ is the combination of sentential variables of the number of leaves n in which the kinds of sentential variables is less than i $(1 \leq i < n)$. $S'_{\text{wfc}}(k)$ denotes the number of kinds of k^{th} degree schemata of wfc,

$$S'_{\text{wfc}}(k) = \sum_{n=1}^{2^{k-1}} B(n) \cdot C_i(n), \; (1 \leq i < n). \tag{4}$$

$C'_i(n)$ is the combination of sentential variables of the number of leaves n in which the kinds of sentential variables is less than i $(1 \leq i < n-1)$. $S''_{\text{wfc}}(k)$ denotes the number of kinds of k^{th} degree schemata of wfc,

$$S''_{\text{wfc}}(k) = \sum_{n=1}^{2^{k-1}} B(n) \cdot C'_i(n), \; (1 \leq i < n-1). \tag{5}$$

The relation between $S_{\text{wfc}}(k)$, $S'_{\text{wfc}}(k)$ and $S''_{\text{wfc}}(k)$ is

$$S''_{\text{wfc}}(k) < S'_{\text{wfc}}(k) < S_{\text{wfc}}(k). \tag{6}$$

Let $\text{Set}^k(\text{wfc})$ the set of k^{th} degree schemata of wfc. $\text{Set}^k(\text{SRP})$ denotes the set of k^{th} degree schemata of wfc satisfied SRP. $\text{Set}^k(\text{CML})$ denotes the set of k^{th} degree logical conditional theorem schemata in CML. $\text{Set}^k(\text{Rc})$ denotes the set of k^{th} degree logical conditional theorem schemata in Rc. The relation of them is as follows,

$$\begin{aligned} \text{Set}^k(\text{Rc}) &\subset \text{Set}^k(\text{CML}), \text{Set}^k(\text{SRP}) \\ &\subset \text{Set}^k(\text{wfc}). \end{aligned} \tag{7}$$

Strong relevance principle SRP limit the kinds of sentential variables in a certain conditional schema. If a conditional schema A, whose the number of leaves is n in represented by a binary tree, is satisfied SRP then the kinds of sentential variables of A is less than or equal to $n/2$ because every sentential variables in A occur at least once as an antecedent part and at least once as a consequent part. On the other hand, if a conditional schema A, whose the number of leaves is n in represented by a binary tree, is a logical theorems of CML then the kinds of sentential variables of A is less than n, because A is not universal true in the sense of CML if the all sentential variables of A is different. If a conditional schema A is a logical theorems of Rc then A is $A \in \text{Set}^k(\text{SRP})$ and $A \in \text{Set}^k(\text{CML})$ (Note this condition is necessary condition but not sufficient condition). In viewpoint of syntax, one of the differences between $\text{Set}^k(\text{CML})$ and $\text{Set}^k(\text{Rc})$ is whether the limit of kinds of sentential variable is strict or not, as same as between $\text{Set}^k(\text{wfc})$ and $\text{Set}^k(\text{SRP})$. Therefore, both the difference between the number of kinds of schemata of wfc and that of wfc satisfied SRP and the difference between the number of kinds of logical conditional theorem schemata in CML and that in Rc have a same tendency to increase the difference as the degree of nested conditionals increases.

In forward reasoning and/or deduction processes, the execution time of forward reasoning becomes longer in polynomial to the increasement amount of the premises and the deduced conclusions [10]. Hence, the forward reasoning based on CML will spend the useless execution time compared with the forward reasoning based on SRL since CML has the large amount of paradoxical conditionals.

5. Concluding remarks

We have investigated the number of paradoxical conditionals in classical mathematical logic by comparing the kinds of schemata of well-formed conditionals with the kinds of schemata of well-formed conditionals satisfied strong relevance principle. From our comparative study, we have been able to analogize the tendency that the number of paradoxical paradoxes in classical mathematical logic increases as the degree of nested conditionals increases. We have therefore showed that classic mathematical logic was quantitatively unsuitable for forward reasoning and/or deduction processes.

In this paper, we focused on implicational paradoxes in classical mathematical logic only. So we did not investigate conjunction-implicational paradoxes and disjunction-implicational paradoxes in classical mathematical logic as well as traditional relevant logics. We think it is possible to prove traditional relevant logics unsuitable for forward reasoning quantitatively by clarifying the number of paradoxical conditionals in traditional relevant logics with comparative study between traditional relevant logics and strong relevant logics like this paper's approach.

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